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A Historical and Critical Review of Mathematical Science

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VOLUME XIX., NUMBER 1

OCTOBER, 1912

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CONTENTS

PAGE

Surfaces of Revolution of Minimum Resistance. By Dr. E. J. MILES - - - - - - - - - - - 1

Shorter Notices - - - - - - - - - - - 12

Notes - - - - - - - - - - - - 33

New Publications - - - - - - - - - - 44

VOLUME XIX., NUMBER 2

NOVEMBER, 1912

Whole No. 212

CONTENTS

PAGE

The Nineteenth Summer Meeting of th American Mathematical Society. By Professor F. N. COLE - - - - 51

A Few Theorems Relating to Sylow Subgroups. By Professor G. A. MILLER - - - - - - - - - 63

Theorems on Functional Equations. By Mr. A. R. SCHWEITZER - - - - - - - - - - - 66

Double Curves of Surfaces Projected from Space of Four Dimensions. By Dr. S. LEFSCHETZ - - - - - 70

Geometrical Optics. By Professor E. B. WILSON - - - 74

Shorter Notices - - - - - - - - - - - 80

Notes - - - - - - - - - - - - 93

New Publications - - - - - - - - - - 102

VOLUME XIX., NUMBER 3

DECEMBER, 1912

Whole No. 213

CONTENTS

PAGE

The Fifth International Congress of Mathematicians, Cambridge, 1912. By Professor VIRGIL SNYDER - - - 107

The Fifth International Congress of Mathematicians. Section I: Arithmetic, Algebra, Analysis. By Professor A. B. FRIZELL - - - - - - - - - 131

Shorter Notices - - - - - - - - - - 146

Notes - - - - - - - - - - - 149

New Publications - - - - - - - - - 156

VOLUME XIX., NUMBER 4

JANUARY, 1913

Whole No. 214

CONTENTS

PAGE

The October Meeting of the American Mathematical Society. By Professor F. N. COLE - - - - - 163

The Twenty-Second Regular Meeting of the San Francisco Section. By Professor T. M. PUTNAM - - - 174

The Fifth International Congress of Mathematicians. Sections I–IV. By Professor VIRGIL SNYDER - - - 175

The Münster Meeting of the Deutsche Mathematiker-Vereinigung. By Professor VIRGIL SNYDER - - - 191

Shorter Notices - - - - - - - - - 197

Notes - - - - - - - - - - - 204

New Publications - - - - - - - - - 212

VOLUME XIX., NUMBER 5

FEBRUARY, 1913

Whole No. 215

CONTENTS

	PAGE
The Sixth Regular Meeting of the Southwestern Section. By Professor J. N. VAN DER VRIES	219
Some Special Boundary Problems in the Theory of Harmonic Functions. By Dr. T. H GRONWALL	227
Note on Fermat's Last Theorem. By Professor R. D. CARMICHAEL	233
Integral Equations. By Professor W. R. LONGLEY	236
An Advance in Theoretical Mechanics. By Professor E. B. Wilson	242
Shorter Notices	246
Notes	264
New Publications	268

VOLUME XIX., NUMBER 6

MARCH, 1913

Whole No. 216

CONTENTS

	PAGE
The Nineteenth Annual Meeting of the American Mathematical Society. By Professor F. N. COLE	275
The Product of Two or More Groups. By Professor G. A. MILLER	303
The Mathematics of Mahāvīrācārya. By Professor D. E. SMITH	310
Shorter Notices	316
Notes	322
New Publications	325

VOLUME XIX., NUMBER 7

APRIL, 1913

Whole No. 217

CONTENTS

PAGE

Some General Aspects of Modern Geometry. By Professor E. J. WILCZYNSKI - - - - - - - - 331

On Certain Non-Linear Integral Equations. By Mr. H. GALAJIKIAN - - - - - - - - - - 342

A Theorem on Asymptotic Series. By Mr. VINCENT C. POOR 346

On Poincaré's Correction to Bruns' Theorem. By Professor W. D. MACMILLAN - - - - - - - - 349

Note on the Groups for Triple Systems. By Miss L. D. CUMMINGS - - - - - - - - - - 355

De Séguier's Theory of Substitution Groups. By Professor G. A. MILLER - - - - - - - - - 356

Wilson's Advanced Calculus. By Professor W. E. BYERLY 360

The Calculus in India. By Professor E. B. WILSON - - 363

Shorter Notices - - - - - - - - - - 367

Notes - - - - - - - - - - - - 375

New Publications - - - - - - - - - 381

VOLUME XIX., NUMBER 8

MAY, 1913

Whole No. 218

CONTENTS

PAGE

The February Meeting of the American Mathematical Society. By Professor F N. COLE - - - - - 387

Three or More Rational Curves Collinearly Related. By Dr. J. E. ROWE - - - - - - - - - 395

Second Note on Fermat's Last Theorem. By Professor R. D. CARMICHAEL - - - - - - - - - 402

An Extension of a Theorem of Painlevé. By Dr. E. H. TAYLOR - - - - - - - - - - 403

Mathematical Physics and Integral Equations. By Dr. W. A. HURWITZ - - - - - - - - - 406

Shorter Notices - - - - - - - - - 411

Notes - - - - - - - - - - - - 429

New Publications - - - - - - - - - 435

VOLUME XIX., NUMBER 9

JUNE, 1913

Whole No. 219

CONTENTS

PAGE

The Spring Meeting of the Chicago Section. By Professor H. E. SLAUGHT - - - - - - - - - 441

Concerning Two Recent Theorems on Implicit Functions. By Dr. L. L. DINES - - - - - - - - 462

Concerning the Property Δ of a Class of Functions. By Professor A. D. PITCHER - - - - - - - - 468

The Asymptotic Form of the Function $\Psi(x)$. By Mr. K. P. WILLIAMS - - - - - - - - - 472

An Erroneous Application of Bayes' Theorem to the Set of Real Numbers. By Dr. E. L. DODD - - - - 479

Shorter Notices - - - - - - - - - - 482

Notes - - - - - - - - - - - - 484

New Publications - - - - - - - - - 489

VOLUME XIX., NUMBER 10

JULY, 1913

Whole No. 220

CONTENTS

PAGE

The April Meeting of the American Mathematical Society. By Professor F. N. COLE - - - - - - - 497

The Twenty-Third Regular Meeting of the San Francisco Section. By Professor W. A. MANNING - - - 517

The Total Variation in the Isoperimetric Problem with Variable End Points. By DR. A. R. CRATHORNE - 519

A Note on Graphical Integration of a Function of a Complex Variable. By Dr. S. D. KILLAM - - - 522

The Unification of Vectorial Notations. By Professor E. B. WILSON - - - - - - - - - - 524

Shorter Notices - - - - - - - - - - 531

Notes - - - - - - - - - - - - 538

New Publications - - - - - - - - - - 541

Twenty-Second Annual List of Published Papers - - 550

Index of Volume XIX - - - - - - - - - 559

BULLETIN OF THE

AMERICAN MATHEMATICAL SOCIETY.

SURFACES OF REVOLUTION OF MINIMUM RESISTANCE.

BY DR. E. J. MILES.

(Read before the Chicago Section of the American Mathematical Society, January 1, 1909.)

THE problem of finding the surface of revolution of minimum resistance may be thought of as the oldest problem of the calculus of variations. A first solution was given by Newton* in 1686. It has since been considered by L'Hospital, August, Silvabelle, Kneser, and others. The results obtained by these writers are based on the Newtonian law of resistance, which states that the resistance R is given by the formula

$$R = f.\ \sin^2 \alpha,$$

where f is the force and α the angle which the line of force makes with the tangent to the surface at the point of application. However, physical experiment does not always verify this law. Especially does it fail† when the angle α is small.

As a result of this, several different laws of resistance have been given; some being derived mathematically, others being stated as verifying experiment. Among these the laws of von Lössl,‡ Duchemin,§ and Kirchhoff‖ have received the greater notice. They are given by the following formulas:

$$R = f \sin \alpha, \quad R = f \frac{2 \sin \alpha}{1 + \sin^2 \alpha}, \quad R = f \frac{(4 + \pi) \sin \alpha}{4 + \pi \sin \alpha}.$$

---

* See Principia Philosophiæ Naturalis, II; Sect. VII, Prop. XXXIV, Scholium.

† For an account of the various physical causes underlying this, see Encyklopädie der mathematischen Wissenschaften, IV, 17, §§ 4, 5, 6.

‡ F. v. Lössl, Die Luftwiderstandsgesetze, Wien, 1896, p. 96.

§ Duchemin, Experimentaluntersuchungen über den Widerstand der Flüssigkeiten, Braunschweig, 1844, p. 101. This law has been verified by Langley, Experiments in Aërodynamics, Washington, 1891, p. 101.

‖ G. Kirchhoff, *Journal für Mathematik*, vol. 70 (1869).

The present paper had its origin in a study of the surfaces of revolution resulting from these separate laws. In each case it was found that the curve determining the surface had properties quite analogous to those of the newtonian curve. This naturally suggested the determination of a general law of resistance having properties which would include the preceding as special cases. Such a law is stated in §1. In this section the discussion of the properties of the curves resulting from this law is given. In the following three sections the theory of the special cases will be taken up in the order mentioned as applications of the general. This general case also includes the newtonian law of resistance and from it the well known results* for this law can be obtained.

§ 1. *Surface of Revolution of Minimum Resistance for a General Law of Resistance.*

In formulating the problem it will be supposed that the surface is formed by the revolution of an arc AB

$$x = x(t), \quad y = y(t) \qquad (t_1 \leqq t \leqq t_2),$$

about the X-axis, where the point A is taken at the origin and B in the interior of the first quadrant. The direction cosines of the tangent are then given by the expressions $x'(t)$ and $y'(t)$. Let this arc be such that $y > 0$ except for $t = t_1$. Further it will be supposed that the body moves in the direction of the negative X-axis with constant velocity.

Consider then the surface resulting from the law of resistance expressed by the formula

$$R = f \cdot \varphi(x', y'), \tag{1}$$

where the function $\varphi(x', y')$ is homogeneous and of dimension zero in x' and y', and f again represents the force. It is readily shown that, aside from a numerical factor, which is independent of the form of the curve, the resistance is given by the definite integral

$$J = \int_{t_1}^{t_2} yy' \varphi(x', y') dt. \tag{2}$$

However from physical considerations† it is necessary to limit

* For a very complete summary of the work done on this classical problem see Bolza, Vorlesungen über Variationsrechung, pp. 407–418.

† See August, *Journal für Mathematik*, vol. 103 (1888), p. 1.

the discussion to curves such that along the arcs AB one has

$$x' \geqq 0, \quad y' \geqq 0.$$

Consequently the problem under consideration may be stated thus:

Among all ordinary curves which go from the origin A to a point B given in the interior of the first quadrant and which besides satisfying the regional restriction

$$y > 0 \quad \text{for} \quad t_1 < t \leqq t_2 \tag{3}$$

also satisfy the slope condition

$$x' \geqq 0, \quad y' \geqq 0 \quad \text{for} \quad t_1 \leqq t \leqq t_2, \tag{4}$$

it is required to find that one which minimizes the integral (2).

Since the restriction has been made that the expression $\varphi(x', y')$ is homogeneous and of dimension zero it follows that it can be written as a function of q, say $\varphi(q)$ for simplicity, where

$$q = x'/y' = \cot \theta, \tag{5}$$

θ being the angle which the tangent makes with the positive X-axis.

The following restriction will now be imposed on the function $\varphi(q)$:

Its derivative $\varphi'(q)$ first decreases continuously from zero to a finite minimum value and then constantly increases to the value zero as q increases from 0 to $+\infty$.

Graphically the function $\varphi'(q)$ is as shown in Fig. 1. If c denotes the value of q, when $\varphi'(q)$ has its minimum, then $\varphi'(q)$ assumes all values between 0 and $\varphi'(c)$ twice.

These conditions are fulfilled in all the special cases to be considered.

Suppose now that an arc of the minimizing curve is considered which is of class C' and such that

$$x' > 0, \quad y' > 0. \tag{6}$$

This arc must then satisfy the Euler differential equation, from which it follows that a first integral is given by the equation

$$yy'\partial\varphi(x', y')/\partial x' \equiv y\varphi'(q) = -a, \tag{7}$$

where a is a constant of integration which must be positive on account of the above conditions. Hence it follows that

$$y = -\frac{a}{\varphi'(q)} = aY(q).$$

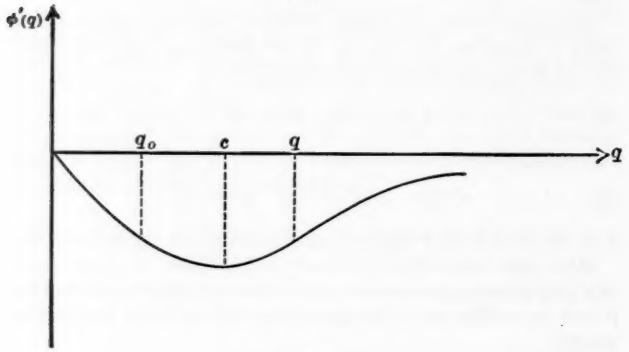

FIG. 1.

But from the relation (5) x' is seen to have the value

$$x' = a\frac{\varphi''(q)}{\varphi'^2(q)}qq'.$$

and therefore x is of the form

$$x = aX(q) + b, \quad X(q) = \int \frac{\varphi''(q)}{\varphi'^2(q)}qdq.$$

The two equations

$$x = aX(q) + b, \quad y = aY(q), \tag{8}$$

furnish the most general solution of the Euler differential equations in terms of the parameter q, when the inequality (6) is satisfied.

But in as much as the general extremal can be obtained from the special curve

$$X = X(q) \equiv \int \frac{\varphi''(q)}{\varphi'^2(q)}qdq, \quad Y = Y(q) \equiv \frac{-1}{\varphi'(q)}, \tag{9}$$

by means of the similarity transformation

(10) $$x = aX + b, \quad y = aY,$$

it is only necessary to study the special curve (9) in order to find the general properties of the extremal curve.

In order to discuss the curve (9) the derivatives will be computed and the expression for the curvature determined under the assumption that both X and Y are infinite for $q = +0$ and $q = +\infty$. It is readily verified that

$$X' = \frac{\varphi''}{\varphi'^2} q, \qquad Y' = \frac{\varphi''}{\varphi'^2},$$

(10) $$X'' = \frac{\varphi'\varphi'' + q\varphi'\varphi''' - 2q\varphi''^2}{\varphi'^3}, \quad Y'' = \frac{\varphi'\varphi''' - 2\varphi''^2}{\varphi'^3},$$

$$X'Y'' - Y'X'' = \frac{\varphi''^2(q)}{\varphi'^4(q)}.$$

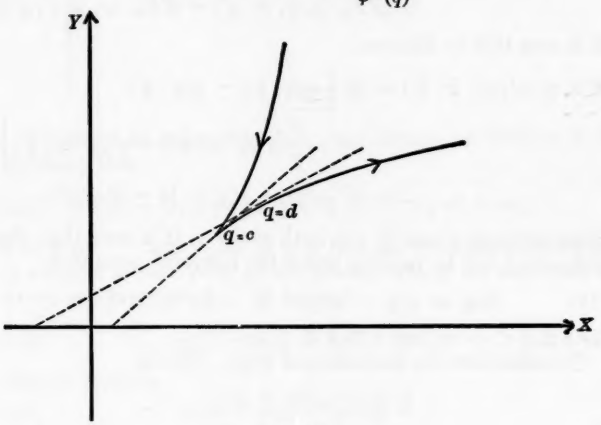

FIG. 2.

Hence the curve has a cusp when $\varphi''(q) = 0$. Let the corresponding value of q be $q = c$. Further, for the range of values

$$0 \leqq q \leqq c$$

the curve has an infinite branch which is convex towards the X-axis, and for

$$c \leqq q < +\infty$$

an infinite branch concave towards the X-axis. The curve is then as shown in Fig. 2.

However the Legendre condition, $F_1 \geqq 0$, shows that the following inequality must hold

$$F_1 \equiv \frac{y}{y'^3}\varphi''(q) \geqq 0.$$

This excludes the possibility of the convex arch of the curve ever furnishing a minimum and thus leaves only the concave arch to be considered.

Further consideration shows that in general the Weierstrass condition excludes a part of this arch. For recalling the definition of the Weierstrass E-function, viz.,

$$E(x, y; x', y'; \bar{x}', \bar{y}') = F(x, y, \bar{x}', \bar{y}') - [\bar{x}'F_{x'}(x, y, x', y') + \bar{y}'F_{y'}(x, y, x', y')]$$

it is seen that in this case

$$E(x, y; x', y'; \bar{x}', \bar{y}') = y\bar{y}'\left\{\varphi(\bar{x}', \bar{y}') - \varphi(x', y') - \frac{\bar{x}'}{\bar{y}'}y'\varphi_{x'}(x', y') - y'\varphi_{y'}(x', y')\right\}$$

$$= y\bar{y}'\{\varphi(\bar{q}) - \varphi(q) - [\bar{q} - q]\varphi'(q)\}.$$

Since however y and $\bar{y}'$ are both positive, it is seen that the E-function will be positive under the following condition:

$$\Phi(\bar{q}) \equiv \varphi(\bar{q}) - \varphi(q) - [\bar{q} - q]\varphi'(q) \geqq 0, \tag{11}$$

for $c \leqq \bar{q} < +\infty$ and $0 \leqq \bar{q} < +\infty$.

Consider now the derivative of $\Phi(q)$. This is

$$\Phi'(\bar{q}) = \varphi'(\bar{q}) - \varphi'(q).$$

From the graph of $\varphi'(q)$ it follows that the second term of the above derivative is always positive, while $\varphi'(\bar{q})$ is always negative, decreasing from zero to a finite negative value when $\bar{q} = c$ and then increasing to 0 as $\bar{q}$ approaches infinity. Hence for any fixed value of q in the range $c \leqq q < +\infty$ the derivative $\Phi'(q)$ is first positive in an interval $0 \leqq \bar{q} < q_0$, where q_0 is the value of $\bar{q}$ which makes $\varphi'(\bar{q}) = \varphi'(q)$. Then in the interval $q_0 < \bar{q} \leqq q$ the derivative is negative. Finally for $q \leqq \bar{q} < +\infty$ the derivative is again positive. Hence

in these intervals the function $\Phi(\bar{q})$ first increases, then decreases to the value 0 and again increases, as shown in Fig. 3.

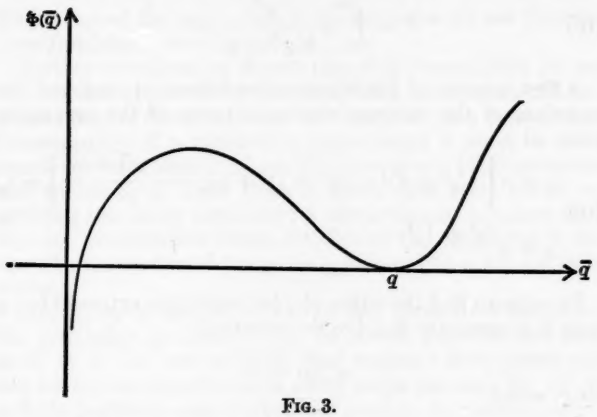

FIG. 3.

Therefore in order that $\Phi(\bar{q})$ may always be positive *it is necessary that*

$$(12) \qquad \varphi(0) - \varphi(q) + q\varphi'(q) \geqq 0 \text{ for } c \leqq q < +\infty;$$

i. e., *if* $\varphi(0) - \varphi(c) + c\varphi'(c) \geqq 0$, *then the Weierstrass condition is satisfied at each point of the concave arch of the curve and a strong minimum results. If however*

$$(13) \qquad \varphi(0) - \varphi(c) + c\varphi'(c) < 0,$$

then the equation

$$(14) \qquad \varphi(0) - \varphi(q) + q\varphi'(q) = 0$$

has one root $d > c$ *and the Weierstrass condition requires that* $q \geqq d$. Only that part of the concave arch beyond the point where $q = d$ furnishes a strong minimum.

§ 2. *The von Lössl Surface.*

In case the law of resistance is taken as that of von Lössl equation (1) becomes

$$R = f \cdot \sin \theta,$$

and the curves defining the surface of revolution must be found among those which minimize the definite integral

$$J = \int_{t_1}^{t_2} y \frac{y'^2}{\sqrt{x'^2 + y'^2}}\, dt. \tag{15}$$

A first integral of Euler's equation follows at once and the equations of the extremal curves in terms of the parameter q are

$$\begin{aligned} x &= a\left[\tfrac{2}{3}(q^2+1)^{\frac{3}{2}} - (q^2+1)^{\frac{1}{2}} + \log\frac{1+\sqrt{1+q^2}}{q}\right] + b, \\ y &= a\frac{(q^2+1)^{\frac{3}{2}}}{q}. \end{aligned} \tag{16}$$

In order to find the value of q for which the extremal has a cusp it is necessary to solve the equation

$$\varphi''(q) = 0$$

for q, where

$$\varphi(q) = \frac{1}{\sqrt{q^2+1}}.$$

This condition requires that $2q^2 - 1 = 0$. So for this problem $c = 1/\sqrt{2}$ and therefore the angle which the cusp tangent makes with the X-axis is

$$\theta = 54^\circ\, 44'.$$

But not all of the concave arch furnishes a minimum, for it can be verified that the inequality (13) is satisfied. Hence in order to find where the minimum actually begins it is necessary to solve the equation resulting from (14), viz.,

$$(q^2+1)^{\frac{3}{2}} - 2(q^2+1) + 1 = 0. \tag{17}$$

This is a cubic of the form

$$x^3 - 2x + 1 \equiv (x-1)(x^2 - x - 1) = 0,$$

where $(q^2+1)^{\frac{1}{2}}$ has been replaced by x. The value $x = 1$ is at once excluded since this would mean that q had the value zero. Solving the quadratic factor for x and taking the positive root, it is at once verified after substituting that q has the

value $q = 1.2719+ \equiv d$ and the corresponding value of θ is

$$\theta = 38^\circ\ 10'+.$$

Hence *beyond the point where θ has this value the von Lösslian curve furnishes a strong minimum.*

Further consideration shows* that if the inequalities (6) are not satisfied on an arc of the minimizing curve, then this arc is a segment of the straight line $x =$ constant or $y =$ constant. Consequently, if a minimizing curve exists it must be composed of a finite number of combinations of von Lösslian curves and segments of these straight lines. But it is found on applying the corner condition for discontinuous solutions that the only combination which satisfies all the conditions is the one where a portion of the y-axis is followed by a von Lösslian curve.

Finally, by a method similar to that given by Kneser† for the newtonian problem it can be shown that *through each point P_2 in the interior of the first quadrant there passes one and but one von Lösslian curve which makes the angle* $38^\circ\ 10'+$ *with the positive x-axis at its initial point in the positive y-axis.*

§ 3. *The Duchemin Surface.*

For Duchemin's law of resistance,

$$R = f \cdot \frac{2 \cdot \sin\theta}{1+\sin^2\theta},$$

aside from a constant factor, the resistance integral is

$$J = \int_{t_1}^{t_2} yy'^2 \frac{(x'^2+y'^2)^{\frac{1}{2}}}{x'^2+2y'^2}\,dt.$$

Expressed in terms of the parameter q, the minimizing curve is then found to have the equations

$$(18)\qquad \begin{aligned} x &= a\left[\tfrac{1}{3}(2q^2-1)(q^2+1)^{\frac{1}{2}} + \frac{6(q^2+1)^{\frac{1}{2}}}{q^2} + 6\log\frac{1+\sqrt{1+q^2}}{q}\right] + b, \\ y &= a\frac{(q^2+2)^2(q^2+1)^{\frac{1}{2}}}{q^3}. \end{aligned}$$

* This statement can be proved by a discussion quite analogous to that found in Bolza, l. c., p. 412.

† *Archiv der Mathematik und Physik*, ser. 3, vol. 2 (1902), p. 273.

Since in this case

$$\varphi(q) = \frac{\sqrt{q^2+1}}{q^2+2}$$

the equation which gives the cusp is

$$2q^4 - 3q^2 - 6 = 0. \tag{19}$$

From this it is seen that

$$q^2 = \frac{3+\sqrt{57}}{4},$$

and therefore the angle which the cuspidal tangent makes with the X-axis is

$$\theta = 31^\circ\ 37'.$$

Again it is at once verified that not all of the concave arch furnishes a minimum. In order to find d it is necessary to solve the equation

$$\frac{1}{2} - \frac{\sqrt{q^2+1}}{q^2+2} - \frac{q^4}{(q^2+2)^2\sqrt{q^2+1}} = 0. \tag{20}$$

After reduction by the substitution of x for $\sqrt{q^2+1}$, equation (20) assumes the form

$$(x-1)^2(x^3 - 2x^2 - 3x - 2) = 0. \tag{21}$$

The value $x = 1$ is again excluded and the positive root of the cubic is found to be

$$x = 3.1515,$$

from which

$$q = 2.9887 \equiv d$$

and therefore

$$\theta = 18^\circ\ 30'.$$

Hence the concave arch of the curve beyond the point where $\theta = 18^\circ\ 30'$ furnishes a minimum.

As in the first example, the most general solution is found to consist of a portion of the y-axis followed by one of the above curves. Furthermore there is always a unique determination of the constants for such curves when only the part of the curve furnishing a strong minimum is considered.

§ 4. *The Kirchhoff Surface.*

According to Kirchhoff, the law of resistance should be

$$R = f \cdot \frac{(4+\pi)\sin\alpha}{4+\pi\sin\alpha}.$$

Neglecting a constant factor which does not affect the shape of the minimizing curve, the resistance integral is found to be

$$J = \int_{t_1}^{t_2} y \frac{y'^2}{\sqrt{x'^2+y'^2}+ey'}\,dt,$$

where $e = \frac{1}{4}\pi$.

The equations of the minimizing curve are then found to be

$$x = a\left[-\tfrac{1}{3}(q^2+1)^{\frac{3}{2}} + (1+2q^2)(q^2+1)^{\frac{1}{2}} + eq^2 + (1+e^2)\log\frac{1+\sqrt{1+q^2}}{q} - 2e\log q\right] + b,$$

$$y = a\frac{\sqrt{1+q^2}(\sqrt{1+q^2}+e)^2}{q}. \tag{22}$$

From the value of $\varphi(q)$ it is found that the equation defining the cusp value of q is

$$2x^3 - 3x - .7854 = 0, \tag{23}$$

where again $\sqrt{1+q^2}$ has been replaced by x. The positive root of this equation is found to be

$$x = 1.34603$$

and hence $q = c = .90098$. Therefore at the cusp the tangent makes an angle

$$\theta = 47^\circ\ 59'$$

with the positive X-axis.

However the inequality (13) is again satisfied and so the root d of equation (14) must be found. For this problem the equation (14) becomes

$$x^3 - 2x^2 - ex + 1 + e \equiv (x-1)\{x^2 - x - (1+e)\} = 0,$$

where x has the same meaning as above. The root $x = 1$ does

not satisfy the previous conditions. The positive root of the quadratic factor is

$$x = 1.9266.$$

From this

$$q \equiv d = 1.64676$$

and therefore

$$\theta = 31^\circ\ 16'.$$

Hence *the concave arch of the curve* (22) *furnishes a minimum for the integral J beyond the point where the tangent makes an angle of* 31° 16′ *with the positive x-axis.*

A statement similar to those in the two preceding sections regarding the most general solution and the determination of the constants holds for this problem.

It should be noted that in neither of the three cases considered is the angle θ corresponding to $q = d$ as large as that of the newtonian problem where $\theta = 45°$. So whether or not the newtonian law fails for small values of the angle α, it is certain that these laws hold *only* for smaller values of the angle than in the newtonian problem.

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SHORTER NOTICES.

Les Systèmes d'Équations aux Dérivées partielles. By CHARLES RIQUIER. Paris, Gauthier-Villars, 1910. xxvii + 590 pp.

DURING the past twenty years Professor Riquier has published a large number of memoirs on the theory of systems of partial differential equations. The main results of his investigations are now made more accessible to mathematicians by incorporating them in a systematic treatise where they are presented from a uniform point of view. The theory of the most general system, containing any number of equations involving any number of functions of any number of independent variables with their partial derivatives of arbitrary order—is naturally extremely difficult, and the author is to be congratulated for the clearness of his treatment. The symbolism and terminology are carefully chosen, the main

theorems are stated in italics, and the proofs, which are sometimes very long—that for the convergence of the integrals of an orthonome system, for example, occupies thirty pages—are divided into numbered parts, so that the reader may catch his breath.

Five of the fourteen chapters into which the work is divided are of general introductory nature, dealing with continuity, series in general and power series, olotrope functions, analytic prolongation, implicit functions. The treatment is confined to ideas and results used in the sequel. These chapters really constitute a fairly complete treatment of the theory of analytic functions. The author follows Méray in general —Weierstrass is not mentioned—but finds it convenient to modify Méray's definition of an olotrope function by requiring that the region in which the function is defined shall be normal (that is, any two points can be connected and every point is interior). The chapter on implicit functions is very elaborate, covering almost fifty pages. From the outset the author considers functions of n variables, which may be real or complex.

The discussion of analytic prolongation (calcul par cheminement) is rather meager. The difficult subject of the prolongation of the solutions of partial differential equations is treated in a few of the author's memoirs, but as the results are incomplete this aspect of the theory is not included in the present treatise.

In order to economize space the author finds it necessary to introduce a large number of new technical terms. For example, a set of n integers $a_1, a_2, \cdots, a_n$ is said to be of taxe inferieure to another set $b_1, b_2, \cdots, b_n$ provided the first non-vanishing difference $a_1 - b_1, a_2 - b_2, \cdots, a_n - b_n$ is negative. Again, if two systems of analytic equations (numerical or differential) are such that each equation of the one system can be obtained by adding multiples of the equations of the other system (the factors being arbitrary analytic functions), the two systems are said to be in correlation multiplicatoire. An index of these terms would have been of distinct value.

The theory of partial differential equations here presented deals exclusively with power series. Its object is to complete the discussion of the Cauchy-Kowalewski existence theorems. The boundary value problem is thus excluded. The first step is to show that for given initial conditions series can be

calculated which formally satisfy the given differential equations; the second step is to show that the series are convergent. This is carried out for orthonome systems (including the Kowalewski type as a very special case) in Chapter VII; the results are extended and simplified in various directions in the following chapters. Besides the general theory the author includes two important applications illustrating the advantage of his methods: the finite deformations of a continuous medium, and the determination of orthogonal curvilinear coordinates, both for n dimensions.

In the final chapter it is shown that, given any system of equations, it is possible by a finite number of steps to find out whether the system is incompatible or compatible; and in the latter event to reduce it to an equivalent completely integrable system whose general solution involves a finite number of arbitraries (functions or constants). The operations involved in the reduction are those that Lie describes as performable, namely, differentiations and eliminations. There are certain unsettled difficulties as to the implicit functions which arise in the reduction. The author states (page 557): "la très grande généralité du problème posé ne nous permettra d'obtenir, dans les raisonnements qui vont suivre, ni une rigueur absolue, ni une précision irreprochable."

In discussing the degree of generality or the degree of infinitude of the solution of partial differential equations, the reviewer has found the following notation very convenient. Let the symbol f_n denote an arbitrary (analytic) function of n independent arguments; in particular f_0 denotes an arbitrary constant. Then the number of particular solutions contained in a general solution involving k_i arbitrary functions of n_i arguments may be denoted by

$$\infty^{\Sigma k_i/n_i}.$$

For example, the number of curves in a plane is $\infty^{1/1}$; the number of curves in space is $\infty^{3/1}$; the number of surfaces in space is $\infty^{1/2}$; the number of surfaces of revolution is $\infty^{1/1+4/0}$, that is, $\infty^{1/1+4}$, since $\infty^{1/0}$ is the same as ∞^1. The notation is capable of abuse and its chief value is heuristic.

Certain fundamental questions of dimensionality suggest themselves. Thus, it is known that the manifolds ∞^n and ∞^m, $n > m$, are distinct in the sense that no continuous one-to-one correspondence can be established between them. This is

in all probability true for the manifolds $\infty^{n/i}$, $\infty^{m/i}$, $n > m$; and for $\infty^{j/i}$, $\infty^{j/j}$, $i > j$; but where is a proof to be found? The question is of interest both for analytic functions and for general continuous functions.

EDWARD KASNER.

Vorlesungen über ausgewählte Gegenstände der Geometrie. Erstes Heft: *Ebene analytische Kurven und zu ihnen gehörige Abbildungen.* Von E. STUDY. Leipzig, Teubner, 1911. 126 pp. M. 4. 80.

As the title indicates, this monograph contains the first part of a series of lectures on a number of selected geometric topics and deals with the geometry in the complex domain, defined as a cartesian plane with ordinary complex numbers as coordinates. Points of such a domain are, for example, imaginary points of intersection of algebraic curves.

In the introduction we find a summary of von Staudt's famous treatment of imaginary elements by elliptic involutions. The disadvantage of this method is that in order to apply it to the solutions of even some very simple problems concerning positional relations a very clumsy apparatus has to be set in motion.

It is therefore desirable to devise a scheme by which problems involving imaginaries can be handled in a simpler and more effective manner. This is exactly what Study accomplishes in a very thorough manner in his valuable monograph.

He starts out by defining the first and second picture (erstes und zweites Bild) of an imaginary point (ξ, η) in a plane. Designating by $\bar{\xi}$ and η the conjugates of ξ and η, the first picture is obtained as the real pair of points of intersections of the left and right handed minimal lines through (ξ, η) and $(\bar{\xi}, \bar{\eta})$. As indicated in a footnote on page 10, this representation is (apparently) due to Laguerre. It is well to state at this point explicitly that as a pioneer in the field of the complex domain Laguerre accomplished as much as any other man and may be justly put on the same level with von Staudt. As early as 1853 Laguerre found the remarkable result that the radian measure of an angle may be defined as the product of $\frac{1}{2}\sqrt{-1} = \frac{1}{2}i$ and the cross-ratio formed by the two sides of the angle and the isotropic lines through its vertex. In two further articles "Sur l'emploi des imaginaires en géométrie,"

which in 1870 also appeared in the *Nouvelles Annales de Mathématiques*, he laid the foundation of the theory which Study so aptly calls geometry in the complex domain.

In this connection also, the author ought not forget to mention the valuable contribution of Darboux in Sur une Classe remarquable de Courbes et de Surfaces,* in which by means of the "points associés," † corresponding to Study's "first picture," he established the focal properties of conics and generalized cassinians and lemniscates.

As the "second picture" (zweites Bild) Study introduces a second couple of real points (Z, W) on the line joining (ξ, η) with $(\bar{\xi}, \bar{\eta})$, which have the same middle point as the first real couple (z, w) and whose distance differs from the purely imaginary distance $\sqrt{(\xi - \bar{\xi})^2 + (\eta - \bar{\eta})^2}$ by a primitive fourth root of unity. According to Study, this idea also is not new and may be traced back to a number of independent investigators. The credit of being the first to make a systematic application of these representations to geometry is according to Study due to Segre.‡ Study however follows an entirely independent path leading to a great number of original views and results.

Without entering into a detailed account of the rest of the subject matter, the nature and scope of the book will perhaps best appear from an explicit statement of a few propositions.

The points, z, w, Z, W form the vertices of a square. Evidently the segment $\overrightarrow{ZW}$ may be obtained from the segment $\overline{zw}$ by turning the latter about its middle point through a positive angle $\frac{1}{2}\pi$. This periodic process which, after repeating it four times, leads to identity is called "positive change of front" (positive Schwenkung) of the couple $\overrightarrow{zw}$. The process

* A. Hermann, Paris, 1st ed. 1873, 2d ed. 1896. Davis, in the University of Nebraska Studies, vol. 10, pp. 1–58 (1910) also gives a method establishing a (1, 1) correspondence between imaginary points and real elements. Recently Mathews in *Proc. London Math. Society*, vol. 10, part 3, pp. 173–190 (1911) has established a similar method.

† It can easily be shown that an imaginary point, its conjugate, and their associated real couple (erstes Bild, points associés) may be represented by the intersections of two orthogonal equilateral hyperbolas. In other words every imaginary point may be represented by two real orthogonal equilateral hyperbolas. The four intersections form an orthogonal quadrangle with the circular points as two of the diagonal points.

‡ "Un nuovo campo di ricerche geometriche," *Atti di Torino*, vol. 25, pp. 35–71, vol. 26, pp. 276–301, 430–457, 592–612 (1899).

which results from three repetitions he calls "negative change of front" (negative Schwenkung). The relations between these couples lead incidentally to some interesting kinematic propositions:

If two opposite vertices of a variable square move with constant velocities on two straight lines, then the same is true of the other vertices.

If two opposite vertices of a variable square move in the same sense on two circles with velocities proportional to the radii, then the same is true of the other vertices. The middle points of the four circles also form a square.

The principal object is, of course, the investigation of analytic curves by means of the real pictures. An analytic curve consists of all complex points and only of such points (with finite coordinates ξ_0, η_0) in whose neighborhood the dependence of ξ and η can be expressed at least by one of the developments

$$\eta - \eta_0 = \mathfrak{P}((\xi - \xi_0)^{\frac{1}{m}}) \quad (m = 1, 2, \cdots),$$

$$\xi - \xi_0 = \mathfrak{P}((\eta - \eta_0)^{\frac{1}{n}}) \quad (n = 1, 2, \cdots),$$

where the $\mathfrak{P}$'s represent convergent power series with positive integral or fractional exponents, without a constant term.

Every complex point of an analytic curve may be represented by either its first or second picture. If this is done for all points of the curve, the first and second pictures of the curve result, whose properties appear from the theorems:

The first picture of an analytic curve is generally any real improper (uneigentlich) conformal transformation $z \rightarrow w$. The second picture is generally a special real proper transformation preserving areas (flächentreue Transformation).

As an analytic thread (Faden) Study defines with Segre the set of ∞^1 complex points whose picture in space of four dimensions is a real branch of an analytic curve. An analytic membrane is a set of ∞^2 complex points whose four-dimensional picture is a real sheet of an analytic surface.

In the succeeding articles the differential elements of arcs in the first and second picture, the points of special behavior of an analytic curve and its pictures are discussed. The ellipse and catenary serve as examples.

Finally a chapter is devoted to the analytic continuation of

the potential function. It contains some valuable suggestions and pertinent remarks on the legitimate use of higher complex number systems. It is shown that the analytic continuation through the bicomplex domain, for instance, cannot yield anything new for the ordinary complex domain which cannot also be obtained by analytic continuation in the latter domain.

On the other hand it is pointed out that certain systems of complex quantities, which do not obey the commutative law of multiplication have been particularly useful for the applications in the theory of groups and in geometry. "Accordingly we must beware of any kind of dogmatism. Science disregards artificial restrictions and acknowledges only its own laws, but no authorities."

In conclusion it is shown that the entire projective geometry in space may be interpreted in the euclidean plane by means of the real point couples or pictures. A number of suggestions are also made, how the methods used by the author may be extended to space.

The little book, on the whole very carefully prepared, thus proves very profitable reading and suggestive for further research.

ARNOLD EMCH.

Vector Analysis. By J. G. COFFIN. Second edition. Wiley and Sons, New York, 1911. 12mo. xxii+262 pages. $2.50.

THE first edition of this book was reviewed in this BULLETIN, volume 17 (1910–11), page 101. The present edition differs little from the first, the important additions occurring in the examples and the appendices. Some few errors have been corrected, and a few statements reworded. Thus, we find the definition of vector now given as follows: "A vector is a directed segment of a straight line on which are distinguished an initial and a terminal point." Exactly what this new definition means, is hard to see. The last clause is superfluous if the main clause means anything, for a segment necessarily has end-points, and if "directed" one end is necessarily initial and the other terminal. Why the end points are specially important is not made clear. Further, the term vector as used in the text does not mean a segment of a straight line, but any one of an infinity of parallel segments of the same currency. It would therefore seem better to

define vector* as "the straight segment from an initial point to a terminal point." The assumption of the equivalence of "free vectors" should then be set forth explicitly.

On page 75, line 13, the three *coefficients* of i, j, k are equal to zero,—there are no three *components* of i, j, k. On page 122, line 15, $\nabla \cdot \mathbf{u}$ should be $\nabla \cdot \mathbf{v}$.

On page 107, despite a minor correction, the paragraph on "Total Derivative of a Function" remains badly confused. The statement "This is the same thing as the directional derivative along $\mathbf{r}_1$ multiplied by dr" is not at all correct, unless it be assumed that $\mathbf{r}$ is to vary only along its own direction. Equation (114) holds for any differential vector $d\mathbf{r}$ and gives the total differential (not *derivative*) of V due to the differential variation of $\mathbf{r}$. This may be taken in the direction $\mathbf{r}_1$ or in any other direction. The author has indeed given on page 103 a diagram in which $d\mathbf{r}$ is marked plainly, and is not in the direction of $\mathbf{r}$. Hence a student is certain to be confused in reading this paragraph.

The additional examples have been well-chosen for the purposes of the text. The additions to the appendix begin on page 240. They distinguish between free and sliding vectors; give some differential geometry of curves including Frenet's formulae; a brief study of the motion of an electron in a magnetic field; further proofs of Stokes' theorem, Gauss' theorem, and two theorems in integration analogous to the divergence theorem. The first proof given of Stokes' theorem on page 249 differs in no essential feature from the more general deduction in Joly's Manual of Quaternions, pages 72–73, and the two theorems on pages 252–253 are given substantially by Tait, Treatise on Quaternions, 3rd edition, § 499, page 384. The second proof of Stokes' theorem on pages 249–250 is given by Tait, § 500, page 385. These two references were surely books on vectors, if not on Vector Analysis.

We desire, in supplying these few corrections and comments, to see a third edition of this successful book even better than the first two.

JAMES BYRNIE SHAW.

*Cf. Appell et Dautheville, Précis de Mécanique rationnelle, p. 1.

Berichte und Mitteilungen der Internationalen mathematischen Unterrichtskommission. Hefte IV–VII. Teubner, Leipzig und Berlin, 1909–1911. Pp. 39–128.

THESE four pamphlets comprise the German reports of the progress and proceedings of the International Commission on the Teaching of Mathematics for the years 1909–1911. They are edited by W. Lietzmann and cover the meeting of delegates at Brussels in August, 1910, and the first international congress on the teaching of mathematics, held at Milan, in September, 1911.

We may sum up the progress which has been made by the commission up to the present year, as given in these reports, by stating that committees, well organized, are now working in twenty-three countries of the world. Under the leadership of Klein these have issued about 100 pamphlets; and many others are in preparation, in manuscript, and in press. As is well known, the plan is comprehensive and covers all fields in which mathematics is taught, from the kindergarten to the research university. Germany and France have contributed most to the literature issued. The United States has also joined in the movement and besides the four reports published in the BULLETIN there have appeared lately five other reports of special committees issued by the United States Bureau of Education. These have already been widely distributed.

The report of the proceedings of the first international congress held at Milan, at which plans were made for a meeting with the fifth international congress of mathematicians held at Cambridge, England, August, 1912, shows that eleven countries were represented by mathematicians of world-wide renown headed by Klein. The special subjects considered dealt with the emphasis placed on various methods used in the teaching of geometry, the teaching of mathematics (pure and applied) to students with major interest in the sciences, and the fusion in the teaching of the various branches of mathematics. These problems were handled in a finished and scientific manner which make the proceedings of many conferences held in this country on similar topics seem crude in comparison. However it must be said that the practical spirit of this country has already adopted and used for some time the so-called reform methods now learnedly discussed by the commission.

The commission expects to complete its labors and report in full to the Congress at Stockholm, in 1916. When the various reports and special articles are published in full, they will give a comprehensive cross-sectional view of the teaching of mathematics as it exists to-day.

ERNEST W. PONZER.

Taschenbuch für Mathematiker und Physiker. Edited by FELIX AUERBACH and RUDOLF ROTHE. 2ter Jahrgang, 1911. Teubner, Leipzig und Berlin. ix + 567 pp.

POCKET and hand books of more or less ambitious size have been in use by engineers, astronomers, and others for a long time. The book under review is the second volume—the first appeared in 1909—of a series of pocket books planned along similar lines for the daily use of mathematicians and physicists. The two sciences are combined in one book because they naturally have much in common, and the 1911 edition also covers in part other fields of science.

It may seem a novel, and certainly an ambitious, project to publish each year a volume which shall remain a reference book and yet not be a duplicate in great part of former volumes issued. It may also be questioned if a series of such separate books would make reference easy. There might also be a question of the number of pockets required. However, while the project might not appeal to an American publisher, it must be said that the firm of Teubner has, as usual, done its part with its usual standard of excellence.

The articles are, as might be expected, very much condensed; in fact in many places definitions and references to books which are authorities in the fields under consideration suffice. The results given are always more than formulas alone, though it would be quite impossible to have treatises on the many subjects considered. Take the case of mathematics, pure and applied. There are 56 articles by various authors covering 180 pages. Among these such subjects as Mengenlehre, quadratic forms, theory of numbers, etc., are included for pure mathematics, while the theories of probability, approximations, and vector analysis are discussed in the section on applied mathematics.

Many parts of mechanics are treated. The tables of logarithms and trigonometric functions given seem totally inadequate. Astronomy is touched upon to the extent of

inserting the calendar for 1911 and an article on the determination of the orbits of planets and comets.

The part devoted to physics starts with a most excellent article by Willy Wien, of Würzburg, which is almost popular in nature and yet so fundamental in treatment as to be worth many a perusal. The various fields of physics are covered by articles and many tables of constants and coefficients, admirable for reference, are included. Fundamental principles expressed in mathematical language abound. In fact, throughout the book the authors, owing to the lack of space, cannot elaborate much.

An article on radioactivity by Greinacher, of Zurich, is of the same character as the one on relativity by Willy Wien.

Several articles on chemical subjects are included; the fields of technology touched upon are those dealing with the theory and design of electrical machinery. All are treated from the viewpoint of the mathematics in the case.

Of the special articles we mention the obituary notice of the late Minkowski by Hilbert and Weyl, of Göttingen, and the article on the present tendencies in the teaching of mathematics in Germany by Lietzmann, of Barmen.

For both the fields of mathematics and physics fairly complete lists of journals, proceedings, recent books, and firms dealing in apparatus are given at the end of the book. A mortuary record for 1909–1910 and a list of teachers in the Hochschulen of Germany are added. Of course, a complete index closes the volume.

ERNEST W. PONZER.

Non-Euclidean Geometry. A critical and historical study of its development. By R. BONOLA. Authorized English translation with additional appendices by H. S. CARSLAW. With an introduction by F. ENRIQUES. Chicago, Open Court Publishing Co., 1912. xii + 268 pp.

THE recent untimely death of Professor Bonola lends unusual interest to this book. In a review of the original Italian edition which appeared in the BULLETIN in 1910 I spoke of the desirability of having "an English edition of so valuable and interesting a work." This want is now well supplied by Professor Carslaw's translation. In a new appendix (the fifth) the translator has also materially improved the book by adding a discussion of a subject which seemed con-

spicuous by its absence from the original, namely, the Klein-Poincaré representation of a non-euclidean plane on a euclidean plane by means of a system of circles orthogonal to a given circle.

In another new appendix (the fourth), the author shows very neatly how to construct projective geometry on the basis of Lobachefskian metrical geometry by adjoining ideal points, lines and planes. This meaning of the word "ideal" is sanctioned by common usage. In the fifth appendix, however, the term "ideal line" is used in a totally different sense, namely, for a circle which images or represents a straight line. This "double entendre" seems perhaps a trifle unfortunate. The translator has produced a very readable and satisfactory English version of the best historical introduction we have to the elements of non-euclidean geometry.

ARTHUR RANUM.

Dr. George Bruce Halsted—Géométrie Rationelle, Traité élémentaire de la Science de l'Espace—Traduction Française par PAUL BARBARIN, avec une preface de C. A. LAISANT. Paris, Gauthier-Villars, 1911. iv + 296 pp.

FROM the time of Farrar and Bowditch a number of French mathematical works have been translated into English, but although several American mathematicians have had their works translated into German, to Dr. Halsted belongs the honor of being the first to be translated into French. Novelties in geometry appeal to the French—witness their creations in connection with the geometry of the triangle, nomography, geometrography, anallagmatic curves and surfaces, and how Méray's somewhat radical work is coming to its own. As could, then, be almost predicted, when the first edition of Professor Halsted's book appeared in 1904 under the title "Rational Geometry, a Text-Book for the Science of Space based on Hilbert's Foundations," it was sympathetically received in France. Barbarin, already well known by his writings on non-euclidean geometry, wrote among other notices (of the first and second editions of Dr. Halsted's book) a ten-page review for Darboux's *Bulletin*.*

In Germany the work was not received so whole-heartedly and Dehn's somewhat vigorously expressed criticisms† (di-

* Sér. 2, vol. 31 (1907), p. 309–319.

† *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Nov., 1904, vol. 13, p. 592–596.

rected chiefly against discussion involving continuity and the style of treatment for a professedly elementary text) seem to have been recognized as just by the author. For, in the "thoroughly revised" and slightly abridged second edition (in which "Based on Hilbert's Foundations" is erased from the title page) changes are made at most points to which Dehn took exception, even in a case (mensuration of a circle) where Professor S. C. Davisson rather took issue* with Dehn.

A detailed account of the content and ideals of the later editions of Professor Halsted's strikingly original book would be merely a repetition of Professor Davisson's review which has already appeared in the BULLETIN.* I shall therefore confine myself to remarks on certain changes introduced into this latest edition and to comment on topics which do not seem to have been referred to in other reviews.†

The French edition is not, strictly speaking, a translation of either of those in English, but is rather a third edition based upon the second English edition. The titles are the same. The heading on page iv, "Preface de l'édition anglaise" is misleading, as the prefaces of both English editions have been combined and translated as that of an original work. The total of numbered paragraphs is the same as in the second English edition,‡ but several are new and revision has taken place throughout. There are 694 exercises instead of 700 and this change was made by leaving out some (e. g., 206, 692) and introducing others (e. g., 674–6). The French edition has only 184 figures while the second had 238. Although the table of contents is given in slightly expanded form, the index has been, unfortunately, left out. To the improvement of the work, Professor Halsted's "Table of Symbols" has been omitted and in the text and problems of the French edition only the ordinary abbreviations and symbols appear. While the English editions had as foot-note to Appendix I: "These proofs are due to my pupil, R. L. Moore, to whom I have been exceptionally indebted throughout the making of the book," the French edition merely gives, "Demonstration due à R. L.

* In the course of a review of the first edition of Professor Halsted's book in the BULLETIN, March, 1905, vol. 11, p. 330–336.

† Hathaway, *Science*, vol. 21 (1905), p. 183; *L'Enseignement Mathématique*, vol. 7 (1905), pp. 160 ff.; *Mathematical Gazette*, vol. 3 (1905), pp. 180–182. Vailati, *Boll. Bibliog. Storia Sc. matem.* (Torino), vol. 8 (1905), pp. 74–77. Eiesland, *Amer. Math. Monthly*, vol. 19 (1912), pp. 91–94. Baker, *Proc. Roy. Soc. Canada*, vol. 12 (1905), pp. 111–126.

‡ That is, counting paragraph 389, which was left out of this edition.

Moore, élève de G. B. Halsted." Barbarin was hard pressed to find single-word equivalents for some of Professor Halsted's terms and he omits such words as "symtra" and "tanchord." In other cases he introduces additional terms; for example, beside the word centroid (first adopted as synonymous with center of gravity by T. S. Davies*) he also gives "barycentre."

Professor Halsted associates names of mathematicians with theorems in perhaps a score of instances; for example: Archimedes (pages 70, 218), Pascal (page 73), Heron (page 110), Pappus (page 157), Euler (page 183), Lexell (page 255), Bordage (page 261), Joachimstal (*sic*, page 262). In practically no case (outside of the historical note on π) is there any indication when the mathematician lived, and in the whole book there is only a single exact reference to another book, namely, to Heiberg's text for Archimedes's axiom. Unless some such information is given in connection with all of the names I see no use in introducing them into a scientific work. And besides, which Heron is meant? Who unacquainted with less prominent French mathematical periodicals ever before heard of Bordage?

But if names are to be introduced, great care should be exercised. On page 278 Professor Halsted gives the example: "If from any point P on the circumcircle of a triangle ABC, PX, PY, PZ be drawn perpendicular to the sides, the points X, Y, Z are collinear on the Simson line of the triangle with respect to P." In the new (1911) edition of the Century Dictionary Professor Halsted gives this same definition. Now, for years it has been known† that this theorem is not to be found in either the published or unpublished work of Simson. On the other hand as William Wallace did enunciate the theorem the line should be known as Wallace's Line,‡ if special denomination be required.

Lexell's theorem is stated: The vertices of spherical triangles of the same angle-sum on the same base are on a circle copolar with the straightest bisecting their sides. I cannot find this in any of Lexell's papers. But the following theorem

* In 1843, *Mathematician*, vol. 1, p. 58. It had been used by Dr. Hey in 1814 to designate another point.

† *Proc. Edinburgh Math. Society*, vol. 9 (1891), p. 83. *Cf.* also Cantor, Vorlesungen über Geschichte der Mathematik, vol. 3 (1901), page 542.

‡ First remarked in 1885 by Dr. Thomas Muir (*Proc. Edinburgh Math. Soc.*, vol. 3, page 104). See my "Historical Note" in the same *Proceedings*, vol. 28 (1910), p. 64.

first discovered by Lexell was published in 1784:* The vertices V of spherical triangles ABV with the same angle-sum on the same base AB are on a certain small circle. Lexell gives† the following construction for this small circle: Through C the middle point of AB draw the great circle CZM, normal to the great circle AB and meeting it again in M; let the great circles AV, BV meet the great circle ABM in O and N respectively. Draw the great circle AXO making with ABO an angle equal to one half of the excess of the given angle-sum over 180°. Through O draw a great circle perpendicular to AXO to meet CZM in P; then with P as pole and distance PO describe a small circle and this is the required locus. In the course of his proof Lexell shows that $PO = PN = PV$. Hence the small circle *NOV is fixed by the points diametrically opposite to the ends of the base.* Steiner therefore incorrectly claims priority‡ in the discovery of this result.

Again, since 1860,§ it is clear that we should no longer refer to the theorem concerning convex polyhedra as the theorem of Euler, even though he enunciated and proved it as early as 1758.‖ For, more than 75 years earlier the same theorem was formulated by Descartes. As Euler was merely a rediscoverer, the theorem should be known as the theorem of Descartes for polyhedra.

On the other hand in connection with the elegant but little known prismoidal two-term formula

$$V = \frac{h}{4}(B + 3S)\text{¶}$$

which "domine toute la mesure des volumes," the name of the Swiss mathematician Hermann Kinkelin who published the formula in 1862** might well be introduced. It was apparently rediscovered and discussed at least twice*** before it appeared

* *Acta academiæ scientiarum imperialis petropolitanæ pro anno* MDCCLXXXI, Pars prior . . . Petropoli, MDCCLXXXIV. "Solutio problematis geometrici ex doctrina sphaericorum," p. 112–126.

† *L. c.*, p. 123.

‡ *Crelle's Journal*, 1827, vol. 2, p. 45.

§ Oeuvres inédites de Descartes, éd. par Foucher de Careil, vol. 2, p. 214 ff.

‖ *Novi Comment. Acad. Sc. petrop.*, vol. 4 (1752–1753), 1758, p. 109–160.

¶ Where h is the height, B the area of either end and S the area of the section two thirds of the distance from that end to the other.

** "Zur Theorie des Prismoides," *Archiv der Mathematik und Physik* (Grunert), vol. 39, p. 185.

*** J.K. Becker, "Einfachste Formel für das Volumen des Prismatoids,"

in Professor Halsted's Elementary Treatise on Mensuration, 1881.* Judging by the preface to the fourth edition of this work and by its denomination as "*New* prismoidal formula" it would appear that the author believed the result to originate with himself.

The historical note on π (pages 151–2) needs to be revised. Ludolf van Ceulen indicated the equivalent of the number π to 35 decimal places not 30.† Vega gave only 136 decimal places correctly, not 140.‡

Except for the lack of an index the French is a great improvement on the English edition. Apart from the figures (e. g., 58, 63, 91, 108, 113 are by no means up to the usual standard set by Gauthier-Villars) the pages are exceedingly attractive and it is to be hoped that a third English edition introducing still further improvements may not be long delayed. The work is full of interest and deals with a discussion of fundamentals in geometry, in an attractive style; and there can be little doubt that the number of universities using Professor Halsted's text in connection with courses on the Foundations of Geometry, will steadily increase.

The Japanese edition of the Rational Geometry which Sommerville lists§ as published in 1911 has not yet (in May, 1912) appeared.

R. C. ARCHIBALD.

Elementary Analysis. By P. F. SMITH and W. A. GRANVILLE. Boston, Ginn and Company, 1910. x + 223 pp.

THE number of textbooks in analytic geometry and calculus is rapidly increasing. But nearly all are intended for the use of students in engineering or for students who intend to specialize in pure or applied mathematics. In view, however, of the recent remarkable development of the natural sciences along mathematical lines, a brief course in analytic geometry and calculus is desirable for the general student who takes one year of mathematics as an elective beyond

Zeitschrift für Mathematik und Physik, vol. 23 (1878), p. 413 (dated Mai, 1877).—T. Sinram, *Archiv der Mathematik und Physik* (Grunert), vol. 63, p. 443 (November, 1878).

* Page 130.

† Bierens de Haan, *Messenger of Math.*, vol. 3 (1874), p. 25; copy of inscription on van Ceulen's tombstone.

‡ G. Vega, Thesaurus Logarithmorum, Leipzig, 1794, p. 633, and W. Shanks, Contributions to Mathematics, London, 1853, p. 86.

§ Bibliography of Non-Euclidean Geometry, 1911.

the required trigonometry and college algebra. The present-day tendency is also to correlate the various branches of mathematics, which, no doubt, is a step in the right direction, provided the idea of correlation is not carried too far.

The little book under consideration is an attempt to solve the problem of writing a textbook, especially adapted to a brief course in analytic geometry and calculus, in which the two subjects are brought into closer contact. The object of the authors is to provide a course which will give the student a broad outlook and which possesses a distinct cultural value. In this attempt the authors has been very successful.

The book is adapted to a course of about seventy-five lessons. The first five chapters deal with the elements of analytic geometry including cartesian coordinates, curves and equations, lines and circles, and curve plotting. No chapter is devoted to conic sections, as is usually done in most textbooks. Examples illustrating the different kinds of conic sections are given, but they appear simply as illustrations of general analytic methods.

In the next seven chapters the fundamental principles of the differential calculus are treated, and the topics considered are differentiation of ordinary functions, tangents, maxima and minima, rates, and differentials. The last four chapters deal with integration of the standard elementary forms, constants of integration, and applications to areas and volumes of solids of revolution.

The treatment throughout the book is very clear and to the point, and the great number of attractive and well-selected problems will be greatly appreciated by the teacher. Great emphasis is laid on graphical representation, and one prominent feature is the discussion of concrete examples before the discussion of a general formula is taken up. It is also very pleasing to note the great care and simplicity with which such subjects as the angle between two lines, the area of a triangle, the limiting value of a function, and several others are treated.

One of the greatest difficulties in writing an introductory textbook in calculus is to give definitions and proofs which a beginner can understand, and at the same time to avoid making any statements which are not correct. It is, of course, impossible to employ the refinements of modern rigor in a book of this kind. The authors make the distinction between

proof and illustration very clear and are very successful in avoiding positive inaccuracies. The only place where the discussion might have been made a little clearer is on page 126, where the number e is defined as the limiting value of $(1+z)^{1/z}$.

The typography and arrangement of matter on the page are excellent, and the book as a whole is very attractive.

JACOB WESTLUND.

Second Course in Algebra. By H. E. HAWKES, W. A. LUBY and F. C. TOUTON. Ginn and Company, 1911. vii + 264 pp.

IN arranging this Second Course to follow the first year's work in algebra the authors have made the student's return to the study of mathematics both interesting and easy. The review of the main features of their First Course in Algebra as given in the earlier pages of this book is of course quite essential. It is presented in a way that leads the reader to more mature and accurate habits of thought; he is frequently shown certain limitations on what he supposed were very easy and familiar operations. From the very beginning of the text there is evident a definite effort to induce him to discriminate accurately and logically. We regret to note later in the book a few unfortunate digressions from the rigorous method of presentation that is so admirable at the beginning. The treatment of linear equations is given a new interest for the reader by the introduction of second and third order determinants. With these of course no proofs are given and little use is made of even the more elementary theorems in determinants. Simply to teach the student the actual use of determinants in the solution of systems of linear equations is certainly the wisest procedure at this stage. Graphs are used quite extensively in the solution of both linear and quadratic equations. A number of very instructive illustrations are given in which the solution of two quadratic equations may be reduced to the solution of a system of linear equations. The straight lines are shown in the graph to pass through the intersections of the conics. This visualizing process ought to give the algebraic manipulation a much more tangible significance. The reviewer does not advocate any proofs with regard to the properties of these conics, with the possible exception of the circle, but he does feel that the straight line and linear equation should be

treated rigorously when the student knows a little plane geometry. Neither in the First Course nor in the present work do the authors offer more than an intuitional explanation of the fact that a first degree equation defines a straight line. This plan of making merely plausible what could easily be proved rigorously is followed in the treatment of maxima and minima. One might question seriously the desirability of trying to find the maximum and minimum values of functions of higher degree than the second by means of elementary algebra. While quadratic functions can be treated rigorously by completing the square (no mention of which is made in the text), the cubics and others will usually present difficulties. When the desired information cannot be obtained accurately by the methods available, we are inclined to doubt the advisability of encouraging the student to guess at results from a picture.

In example 23, page 165, the length of time required to reduce the velocity 2 feet per second should be mentioned. The word "therefore," line 4, page 172, is not justified by the statements which precede it; the theorem in question is not proved for the general case. The word "limit" can scarcely be used with propriety on page 173 when it is not defined until page 176. The expression "about to stop" in example 17, page 173, is too inexact to merit serious criticism. The authors are to be commended for the emphasis placed on the exponential nature of logarithms. The chapter is arranged so as to teach the student to handle logarithmic and exponential equations with equal readiness and to change from one to the other with ease and certainty. The treatment of complex numbers is careful and instructive. That quite erroneous results may be obtained by careless multiplication and division of imaginaries is emphasized and a number of excellent illustrations are given.

J. V. McKelvey.

Le Calcul mécanique. Par L. Jacob. Paris, Doin et Fils, 1911. xvi+412 pp. with 184 figures in the text. 5 fr.

This concise presentation of mechanical calculators is the more valuable and interesting to the reader because of the author's style and systematic treatment. The scientific classification follows that adopted in the conferences in 1893 at the Conservatoire des Arts et Métiers held by M. d'Ocagne

and reported in the volume entitled "Calcul simplifié." The several chapters describe instruments having a common purpose or having the same purpose and involving a common mechanical principle. At the end of each chapter there is given an almost complete chronological list of the instruments of the type described therein, with the names of inventors, dates, and frequently a remark to indicate the special characteristic. Frequent references are made to the articles by R. Mehmke and M. d'Ocagne respectively in the German and French editions of the Encyclopedia of Mathematical Sciences (French: tome I, volume 4, fascicule 2), which follow the same classification and supplement the volume under review.

There are treated instruments for the solution of problems of arithmetic, of algebra, and of analysis. These three broad types are further divided into numerous groups and classes, each with its history and development, with a description of the principle involved, a well-marked figure and enough detail, in many instances, at least, to enable even the amateur mechanician to make a similar instrument or machine.

CHARLES C. GROVE.

Mathematische Theorie der astronomischen Finsternisse. By P. SCHWAHN. Leipzig und Berlin, B. G. Teubner, 1910. 128 pp. + 20 figures in the text.

THIS very readable book forms part 8 of Jahnke's "Mathematisch-physikalische Schriften für Ingenieure und Studierende." Its aim is to give to students of natural sciences at large a clear and simple presentation of the theory of the eclipses of the Moon and the Sun, the passages of Mercury and Venus over the disc of the Sun, and the occultations of stars by the Moon. The theory of Bessel, on account of its inherent elegance was certainly best adapted for this purpose and the author has done well to select it in preference to those of Chauvenet and Wollaston. Since the book was not intended to serve as a guide to the astronomer at an almanac office, the author has introduced simplifications, wherever this could be done without injury to the final object the book was written for. The size of the book and the clear presentation of the material will make it a welcome gift to the professional astronomer, especially when reference to Bessel's original or to Chauvenet's more lengthy presentation of Hansen's method is not required. The publishing house of B. G.

Teubner deserves great credit for issuing the small library of science texts under Professor Jahnke's able editorship. It is believed that a similar undertaking in this country would be of decided benefit to our colleges and high schools, where often the lack of proper information is likely to turn valuable men from the realm of science. A glance at the list of authors of the Jahnke-Teubner series will show that in Germany the professor of the gymnasium is considered as preeminently fitted for this kind of work, since his daily contact with the student who has not yet specialized in a given field makes him for these intermediate texts a most excellent interpreter.

KURT LAVES.

Mécanique sociale. By SP. C. HARET. Paris and Bucharest, Gauthier-Villars, 1910. 256 pp.

THIS book, which presents rather entertaining reading to the student of mechanics tries to describe in mathematical language the phenomena of sociology. The social body—an assemblage of individuals, who act on each other and are subject besides to exterior forces of intellectual, economical and moral character—is considered for simplicity to be of invariable form, for a given length of time. Each individual is characterized by three rectangular coordinates, of which x stands to show the economic, y the intellectual, z the moral asset of the individual. On this basis it is not difficult to write out the differential equations of motion in this "social space." The author does not feel any hesitation in transcribing into this social space the axioms and theorems of rational mechanics, but it need not be pointed out to a mathematical audience, that the difficulties here encountered are enormous. The material in the hands of the sociologist today is hardly in such a shape that the mathematician may properly step in with such an ambitious-looking set of differential equations as our author does, and deduce from them statements which have a meaning only in the realm of physics, as far at least as we know at present.

KURT LAVES.

NOTES.

THE July number (volume 13, number 3) of the *Transactions of the American Mathematical Society* contains the following papers: "Quaternion developments with applications," by J. B. SHAW; "Theory of finite algebras," by H. S. VANDIVER; "On the degree of convergence of the development of a continuous function according to Legendre's polynomials," by DUNHAM JACKSON; "Functional differential geometry," by LOUIS INGOLD; "On the extension of a theorem of Poincaré for difference-equations," by E. B. VAN VLECK; "One-parameter projective groups and the classification of collineations," by E. B. VAN VLECK; "Bicombinants of the rational plane quartic and combinants of the rational plane quintic," by J. E. ROWE.

THE July number (volume 34, number 3) of the *American Journal of Mathematics* contains: "Minimal surfaces in euclidean four space," by L. P. EISENHART; "A contribution to the foundations of Fréchet's Calcul Fonctionel," by T. H. HILDEBRANDT; "On the perspective Jonquières involutions associated with the (2, 1) ternary correspondence," by P. P. BOYD; "Some geometrical theorems connected with Laplace's equation and the equation of wave motion," by H. BATEMAN.

THE June number (volume 13, number 4) of the *Annals of Mathematics* contains: "On the rectilinear congruence realizing a circular transformation of one plane into another," by A. EMCH; "On Duhamel's theorem," by R. L. MOORE; "On linear equations with an infinite number of variables," by MAXIME BÔCHER and L. BRAND; "On the theory of correlation with special reference to certain significant loci on the plane of distribution in the case of normal correlation," by H. L. RIETZ.

THE Society for the Promotion of Engineering Education has reprinted as a separate volume the Syllabus of Mathematics compiled by its committee on the teaching of mathematics to students of engineering. The syllabus covers elementary algebra, geometry and mensuration, plane trigonometry, analytic geometry, calculus, and complex quantities.

The volume, which is neatly bound, is sold at cost price, seventy-five cents. Orders should be addressed to the Secretary of the society, Professor H. H. Norris, Cornell University, Ithaca, N. Y.

THE publishing house of Mattei and Co. in Pavia announces that the following books are in press: C. BURALI-FORTI and R. MARCOLONGO, "Analyse vectorielle générale, II.: Applications physico-mécaniques"; T. BOGGIO, "Analyse vectorielle générale: III. Hydrodynamique"; G. VIVANTI, "Esercizi di analisi infinitesimale."

THE publishing house of B. G. Teubner in Leipzig and Berlin announces that the following books are in press: A. BRILL, "Einführung in das Relativitätsprinzip"; R. MEHMKE, "Vorlesungen über Punkt- und Vektorenrechnung"; G. HESSENBERG, "Transzendenz von e und π"; E. R. NEUMANN, "Beiträge zu einzelnen Fragen der höheren Potentialtheorie"; O. PERRON, "Lehre von den Kettenbrüchen"; R. FRICKE and F. KLEIN, "Vorlesungen über automorphe Funktionen" (concluding fascicule); L. LEWENI, "Konforme Abbildung"; G. LORIA, "Vorlesungen über darstellende Geometrie, II"; H. E. TIMERDING, "Die Fallgesetze"; M. ZACHARIAS, "Einführung in die projective Geometrie."

THE annual list of American doctorates published in *Science* presents for the academic year 1911–1912, 492 names, of which 273 are credited to the sciences. The following 22 successful candidates offered mathematics as major subject (the titles of the theses are appended): H. DEF. ARNOLD, Chicago, "Limitations imposed by slip and inertia terms upon Stokes's law for the motion of spheres through liquids"; S. E. BRASEFIELD, Cornell, "A study of certain force fields"; E. W. CHITTENDEN, Chicago, "Infinite developments and the composition property $(K_{12}B_1)_*$ in general analysis"; A. L. DANIELS, Yale, "On the librations of bodies whose periods are one third that of the disturbing body"; W. W. DENTON, Illinois, "Projective differential geometry of developable surfaces"; L. L. DINES, Chicago, "The highest common factor of a system of polynomials with an application to implicit functions"; C. A. FISCHER,

Chicago, "Some contributions to the theory of functions of lines"; T. FORT, Harvard, "Problems connected with the linear difference equations of the second order with special reference to equations with periodic coefficients"; Miss C. B. HENNEL, Indiana, "Certain transformations and invariants connected with difference equations and other functional equations"; C. G. P. KUSCHKE, California, "The abelian equations of the 10th degree, irreducible in a given rational domain"; J. LIPKE, Columbia, "Natural families of curves in a general curved space of n dimensions"; F. M. MORGAN, Cornell, "Involutorial transformations"; R. E. ROOT, Chicago, "Iterated limits in general analysis"; L. P. SICELOFF, Columbia, "Simple groups from order 2001 to order 3640"; W. M. SMITH, Columbia, "Simply infinite systems of plane curves. A study of isogonals, equitangentials and other families of trajectories"; C. T. SULLIVAN, Chicago, "Properties of surfaces whose asymptotic lines belong to linear complexes"; J. I. TRACEY, Johns Hopkins, "Researches on the rational quintic"; E. E. WHITFORD, Columbia, "The Pell equation"; H. R. WILLARD, Yale, "On a family of oscillating orbits of short period (with a chart)"; A. H. WILSON, Chicago, "The canonical types of nets of quadratic forms in the Galois field of order p^n"; R. M. WINGER, Johns Hopkins, "On self-projective rational curves of the fourth and fifth order"; B. M. WOODS, California, "A discussion by synthetic methods of two projective pencils of conics."

THE following university courses in mathematics will be offered at German universities during the winter semester:

UNIVERSITY OF BONN.—By Professor E. STUDY: Differential and integral calculus, II, four hours; Seminar, two hours; Colloquium on the theory of invariants, one hour.—By Professor F. LONDON: Descriptive geometry, II, with exercises, four hours; Analytical mechanics, four hours.—By Professor F. HAUSDORFF: Analytic geometry, four hours; Linear differential equations, two hours.—By Dr. J. O. MÜLLER: Introduction to algebra and the theory of determinants, three hours; Exercises in the calculus, one hour.

UNIVERSITY OF HALLE.—By Professor A. WANGERIN: Analytic geometry of space, three hours; Partial differential

equations of mathematical physics, four hours; Selected chapters from the theory of surfaces, one hour; Seminar, two hours.—By Professor A. GUTZMER: Integral calculus, with exercises, four hours; Descriptive geometry, with exercises, four hours; Seminar, two hours.—By Professor V. EBERHARD: Algebraic equations, four hours; Colloquium, two hours.

UNIVERSITY OF LEIPZIG.—By Professor O. HÖLDER: Mechanics, five hours; Foundations of arithmetic, two hours; Seminar, two hours.—By Professor K. ROHN: Applications of the calculus to surfaces and space curves, four hours; Descriptive geometry, II, with exercises, four hours.—By Professor G. HERGLOTZ: Differential and integral calculus, five hours; Calculus of variations, two hours; Seminar, two hours.—By Professor P. KOEBE: Analytic geometry of space and determinants, with exercises, five hours; Theory of functions, II, two hours; The problem of space, one hour.—By Dr. G. KÖNIG: Introduction to the theory of numbers, II, two hours; Exercises in mechanics, two hours; Exercises in descriptive geometry, two hours.

UNIVERSITY OF MUNICH.—By Professor A. PRINGSHEIM: Algebra, four hours; Theory of functions, four hours.—By Professor F. LINDEMANN: Foundations of geometry, two hours; Plane analytic geometry, four hours; Analytic mechanics, four hours; Seminar.—By Professor A. VOSS: Theory of algebraic curves, three hours; Differential calculus, four hours; Seminar.—By Professor H. BRUNN: Elements of higher mathematics and descriptive geometry, four hours.—By Professor G. HARTOGS: Differential calculus, two hours; Integral calculus, four hours; Exercises, one hour.—By Dr. K. BÖHM: Theory of integral equations, two hours; Statistics, two hours; Life insurance, four hours.

UNIVERSITY OF STRASSBURG.—By Professor H. WEBER: Calculus, four hours; Elliptic functions, two hours; Geometry of numbers, one hour; Seminar.—By Professor F. SCHUR: Projective geometry, four hours; Foundations of geometry, two hours; Seminar.—By Professor J. WELLSTEIN: Algebraic functions and their integrals, four hours; Riemann surfaces, two hours.—By Professor R. v. MISES: Analytic geometry, four hours; Integral equations, two hours; Seminar.—By Professor P. EPSTEIN: Theory of numbers, three hours.—By

Professor M. SIMON: History of mathematics in antiquity, three hours.—By Dr. A. SPEISER: Differential equations and transformation groups, two hours.

THE following advanced courses in mathematics are offered at the Italian universities during the academic year 1912–1913. Courses in algebra, analytic geometry, projective and descriptive geometry, and elementary courses in the calculus, mechanics, astronomy, and geodesy are not included:

UNIVERSITY OF BOLOGNA.—By Professor P. BURGATTI: Troubled motion of the planets, classical and modern theories, three hours.—By Professor L. DONATI: Thermodynamics and its correlations to the electrodynamics of moving bodies, three hours.—By Professor F. ENRIQUES: Theory of algebraic functions, three hours.—By Professor S. PINCHERLE: Advanced calculus, elementary theory of integral equations, three hours.

UNIVERSITY OF CATANIA.—By Professor M. DEFRANCHIS: Hyperelliptic surfaces, four hours.—By Professor G. LAURICELLA: Orthogonal functions, applications to developments and integral equations, three hours.—By Professor G. PENNACCHIETTI: Elliptic functions with particular regard to their applications in mechanics, four hours.—By Professor C. SEVERINI: Integral equations, four hours.

UNIVERSITY OF GENOA.—By Professor E. E. LEVI: Differential equations, four hours.—By Professor G. LORIA: Differential geometry of curves and surfaces, three hours.—By Professor O. TEDONE: Acoustics, three hours.

UNIVERSITY OF NAPLES.—By Professor F. AMODEO: History of mathematics from Galileo to Newton, three hours.—By Professor A. DEL RE: General analysis of Grassmann with applications to geometry and mechanics, four and one-half hours.—By Professor G. GALLUCI: Theory of configurations, two hours.—By Professor R. MARCOLONGO: Theory of elasticity, vibrations of elastic bodies, elastic membranes, three hours.—By Professor D. MONTESANO: General theory of surfaces, surfaces of the third order, four and one-half hours; Geometry of the imaginary elements, three hours.—By Professor E. PASCAL: Riemann surfaces and functions on them, three hours.—By Professor L. PINTO: Propagation of heat, four and one-half hours.

UNIVERSITY OF PADUA.—By Professor F. D'ARCAIS: Functions of a complex variable with classical applications, four hours.—By Professor U. CISOTTI: Mathematical theory of elasticity with technical applications, three hours.—By Professor P. GAZZANIGA: Theory of numbers, three hours.—By Professor T. LEVI-CIVITA: Analytical mechanics with applications to thermodynamics and relativity, four and one-half hours.—By Professor G. RICCI; Potential theory with applications, four hours.—By Professor F. SEVERI: Non-euclidean geometry, four hours.—By Professor G. VERONESE: Principles of elementary geometry, three hours.

UNIVERSITY OF PALERMO.—By Professor G. BAGNERA: Theory of integral equations and their applications in analysis, three hours.—By Professor M. GEBBIA: Vector fields, theory of electricity and magnetism, four and one-half hours.—By Professor G. B. GUCCIA: General theory of algebraic curves and surfaces, four and one-half hours.—By Professor A. VENTURI: General and special perturbations of the planets, eulerian cycle and movements of the terrestrial pole, three hours.

UNIVERSITY OF PAVIA.—By Professor L. BERZOLARI: Differential geometry, three hours.—By Professor F. GERBALDI: Functions of a complex variable, elliptic functions, three hours.—By Professor G. VIVANTI: Theory of contact transformations, three hours.—By ————: Mathematical physics, three hours.

UNIVERSITY OF PISA.—By Professor E. BERTINI: Projective geometry of hyperspaces, three hours.—By Professor L. BIANCHI: Functions of a complex variable, elliptic functions, four and one-half hours.—By Professor U. DINI: Fourier series, development of an arbitrary function of a real variable in terms of the integrals of a linear differential equation of the second order, four and one-half hours.—By Professor G. A. MAGGI: Analytic dynamics, electronic theory of the electromagnetic field, four and one-half hours.—By Professor G. PIZZETTI: General spherical astronomy, determination of orbits, principles of the theory of perturbations, four and one-half hours.

UNIVERSITY OF ROME.—By Professor L. BISCONCINI: Geometrical applications of the calculus, three hours.—By

Professor G. CASTELNUOVO: Differential geometry, three hours. —By Professor V. VOLTERRA: Differential equations of mathematical physics, three hours; Functions which depend on other functions, with applications to mechanics, three hours.—By ————: Higher analysis, three hours.

UNIVERSITY OF TURIN.—By Professor T. BOGGIO: Hydrodynamics, three hours.—By Professor G. FUBINI: Euclidean and non-euclidean geometry, congruent partitions of plane and space, functions of a complex variable, automorphic functions, three hours.—By Professor G. SANNIA: Differential geometry, two hours.—By Professor C. SEGRE: Geometric entities connected with linear systems of curves and surfaces of the second order, three hours.—By Professor C. SOMIGLIANA: Theory of elasticity with applications, three hours.

THE Prussian academy of sciences has advanced M800 toward the preparation of a new edition of Poggendorff's biographisch-literarisches Lexikon.

THE Paris academy of sciences has awarded the Poncelet prize (2,000 fr.) in pure mathematics to Professor E. MAILLET, of the Ecole des Ponts et Chaussées, for his contributions to mathematics. The Francoeur prize (1,000 fr.) was awarded to the estate of the late Dr. E. LEMOINE for the totality of his work in mathematics.

DR. ALFRED ACKERMANN, senior member of the publishing house of B. G. Teubner, has presented the sum of M20,000 to the University of Leipzig, to establish the "Alfred Ackermann-Teubner memorial prize for the promotion of mathematical sciences," to be awarded under the following conditions.

(1) For the present a cash prize of M1000 shall be awarded in the year 1914, and at intervals of two years thereafter.

(2) The unused interest shall be applied to the original capital until it reaches M60000.

(3) If circumstances permit, the prize shall be awarded annually from that time, and may be increased to correspond to the income.

(4) The prize shall be awarded in the following subjects, as defined in the Encyklopädie, in sequence: 1, history,

philosophy, didactics and pedagogy; 2, arithmetic and algebra; 3, mechanics; 4, mathematical physics; 5, analysis; 6, astronomy and the theory of errors; 7, geometry; 8, applied mathematics.

(5) While not excluding others, German competitors are to be considered first.

PROFESSOR P. DUHEM, of the University of Bordeaux, has been elected a member of the royal institute of Venice.

PROFESSOR L. HEFFTER, of the University of Freiburg, has been elected to membership in the academy of sciences of Halle.

CAMBRIDGE University has conferred its honorary doctorate of science on Professor E. PICARD of the University of Paris.

PROFESSOR O. BOLZA, of the University of Freiburg, has been elected to membership in the academy of sciences of Halle.

THE Italian society of sciences (the forty) has awarded its gold medal to Professor E. ALMANSI, of the University of Pavia, for his researches in mechanics and mathematical physics.

PROFESSOR O. HENRICI, of the City and Guilds Engineering College, London, has retired from active service. In recognition of his association with the institution, a gold medal, to be known as the Henrici medal, will be awarded annually for proficiency in mathematics.

PROFESSOR E. ALMANSI, of the University of Pavia, has accepted a professorship of rational mechanics at the University of Rome.

S. P. PEÑALVER Y BACHILLER has been appointed professor of higher analysis in the University of Sevilla.

DR. J. KOUNOWSKI has been appointed docent in geometry at the Bohemian technical school at Prague.

DR. M. RYCHLIK has been appointed docent in mathematics at the Bohemian University of Prague.

PROFESSOR G. KOHN, of the University of Vienna, has been promoted to a full professorship of mathematics.

PROFESSOR S. FINSTERWALDER has been promoted to the professorship of descriptive geometry of the technical school of Munich, held by Professor L. Burmester, who has retired.

DR. A. N. WHITEHEAD has been appointed reader in geometry in University College, University of London.

DR. M. POWER, of the University of Dublin, has been appointed professor of mathematics at the University College of Galway.

PROFESSOR E. T. WHITTAKER has been called to the University of Edinburgh to succeed the late Professor G. Chrystal in the chair of mathematics.

MR. W. D. EVANS, lecturer in mathematics at Hartley College, has been appointed Richardson lecturer in mathematics at the University of Manchester.

AT the Bedford College for Women, Dr. H. B. HEYWOOD has been appointed an assistant lecturer in mathematics and Miss M. LONG has been appointed an assistant in mathematics.

DR. G. SCORZA, of the University of Palermo, has been appointed professor of projective and descriptive geometry at the University of Cagliari.

PROFESSOR K. DOEHLEMANN, of the University of Munich, has accepted a full professorship of mathematics at the technical school of Munich.

MR. C. L. LANG has been appointed professor of mathematics and physics at the College of Agriculture, University of Porto Rico.

THE formal ceremonies attending the opening of the Rice Institute, at Houston, Texas, will take place on October 10–12. The inaugural exercises will include courses of lectures by distinguished foreign scientists, in mathematics Professors BOREL and VOLTERRA. The following appointments have been made in the department of mathematics: Professor, President E. O. LOVETT; assistant professor, Dr. G. C. EVANS; research associate in applied mathematics, P. J. DANIELL, M.A. (Cambridge).

MR. E. P. R. DUVAL, of Princeton University, has been appointed assistant professor of mathematics in the University of Kansas.

PROFESSOR R. D. CARMICHAEL, of the University of Indiana, has been promoted to an associate professorship of mathematics.

AT Brown University Dr. R. G. D. RICHARDSON has been promoted to an associate professorship of mathematics.

DR. E. W. SHELDON has been promoted to the professorship of mathematics in the University of Alberta.

DR. W. M. SMITH, of Lafayette College, has been appointed assistant professor of mathematics in the University of Oregon.

MR. R. B. STONE has been appointed instructor in mathematics at Purdue University.

DR. E. T. BELL, of Columbia University, has been appointed instructor in mathematics at the University of Washington.

PROFESSOR O. D. KELLOGG, of the University of Missouri, is spending a half-year's leave of absence abroad.

PROFESSOR M. W. HASKELL, of the University of California, has received a half-year's leave of absence, which he will spend abroad.

PROFESSOR GUSTAVE LEGRAS, of the College of the City of New York, died July 25 at the age of fifty-four years. He had been associate professor of mathematics in the College since 1884, and was one of the earliest members of the American Mathematical Society, having entered in December, 1889.

PROFESSOR E. L. RICHARDS, emeritus professor of mathematics in Yale University since 1906, died August 6 at the age of 74 years.

THE death is announced of Professor J. L. PTAŠICKIJ, professor of mathematics at the University of St. Petersburg.

PROFESSOR K. v. DER MÜHLL, of the University of Basel, died May 8, at the age of 70 years. He had been an associate editor of the *Mathematische Annalen* since 1873.

PROFESSOR J. HENRI POINCARÉ, of the University of Paris, died July 17, 1912, from an embolism after a two weeks' illness. He was born in Nancy, April 29, 1854, received the degree of doctor of science from the University of Paris in 1879, was elected to the academy of sciences in 1887, and to the French academy in 1908, besides holding membership in nearly all the learned societies of Europe. In 1885 he was awarded the Poncelet prize, and in 1896 the Reynaud prize, both by the academy of sciences of Paris; moreover, he received the King Oscar II prize in 1889, the gold medal of the royal astronomical society in 1900, the Sylvester medal in 1901, the Lobachevsky prize in 1904, the first award of the Bolyai prize in 1905, and the gold medal of the French association for the advancement of science in 1909. Regarding his contributions to the theory of functions, to the foundations of geometry, to the theory of relativity, and his influence in mathematical thought in general, appropriate mention will be made in another place.

CATALOGUE of second hand mathematical books: Mayer & Muller, Prinz Louis Ferdinand Strasse 2, Berlin, catalogue 268, about 3000 titles.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BAKER (W.). The calculus for beginners. London, Bell, 1912. 8vo. 174 pp. 3s.

BARIDON (P.). See BURALI-FORTI (C.).

BURALI-FORTI (C.) et MARCOLONGO (R.). Analyse vectorielle générale. I: Transformations linéaires. Traduit de l'italien par P. Baridon. Pavia, Mattei, 1912. 8vo. L. 6.00

DARBOUX (G.). Éloges académiques et discours. Volume publié par le comité du jubilé scientifique de M. Gaston Darboux. Paris, Hermann, 1912. 8vo. 525 pp.

DÖLP (H.). Grundzüge und Aufgaben der Differential- und Integralrechnung nebst den Resultaten. Neu bearbeitet von E. Netto. 13te Auflage. Giessen, Töpelmann, 1912. 8vo. 3+216 pp. M. 1.80

FERMAT. Oeuvres, publieés par les soins de P. Tannery et C. Henry. Tome IV: Compléments, par C. Henry. Supplément a la correspondence. Appendice. Notes et Tables. Paris, Gauthier-Villars, 1912. 4to. Fr. 14.00

FORSYTH (A. R.). Lehrbuch der Differentialgleichungen. Mit den Auflösungen der Aufgaben von H. Maser. 2te autorisierte Auflage. Nach der 3ten dcs englischen Originals besorgt und mit einem Anhang von Zusätzen versehen von W. Jacobsthal. Braunschweig, Vieweg, 1912. 8vo. 22+921 pp. M. 21.50

GAUSS (C. F.). Allgemeine Flächentheorie. (Disquisitiones generales circa superficies curvas.) (1827.) Deutsch herausgegeben von A. Wangerin. 4te Auflage. (Ostwald's Klassiker. Neue Auflage. Nr. 5.) Leipzig, Engelmann, 1912. 8vo. 64 pp. M. 0.80

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HEIBERG (J. L.). See TANNERY (P.).

HENRY (C.). See FERMAT.

HESS (W.). Beweis dass die Gleichung $c^{2d+1} = a^{2d+1} + b^{2d+1}$ unlösbar ist, wenn a, b, c, d ganze Zahlen > Null. Dresden, Köhler, 1912. 8vo. 4 pp. M. 0.50

HOBSON (E. W.). Treatise on plane trigonometry. 3d edition. Cambridge, University Press, 1912. 8vo. 12s.

HULBURT (L. S.). Differential and integral calculus; an introductory course for college and engineering schools. New York, Longmans. 8vo. 18+481 pp. Cloth. $2.25

JACOBSTHAL (W.). See FORSYTH (A. R.).

LEWENT (L.). Konforme Abbildung. (Mathematisch-physikalische Schriften für Ingenieure und Studierende.) Leipzig, Teubner, 1912. 8vo, 112 pp.

MARCOLONGO (R.). See BURALI-FORTI (C.).

MASER (H.). See FORSYTH (A. R.).

MERCER (J. W.). Exercises from the Calculus for Beginners. Cambridge, University Press, 1912. 8vo. 168 pp. 3s.

MILNE (J. J.). Elementary treatise on cross-ratio geometry; with historical notes. New York, Putnam, 1912. 8vo. 24+288 pp. $2.00

MONTEIL (P. L.). Théorie du point. Géométrie rectiligne et curviligne. Édition nouvelle. Cette Ouvrage contient les solutions de la quadrature du cercle, la trésection de l'angle, et la proposition de Fermat. Paris, Dunod et Pinat, 1912. 4to. 5+140 pp.

NETTO (E.). See DÖLP (H.).

NEUE BÜCHER über Naturwissenschaften und Mathematik. (Die Neuigkeiten des deutschen Buchhandels.) Mitgeteilt Frühjahr 1912. Leipzig, Hinrichs. 8vo. pp. 1–22. M. 0.30

SALOMON (G.). Essais de magie arithmétique polygonale. Paris, Gauthier-Villars, 1912. 8vo. 24 pp. Fr. 1.50

TANNERY (P.). Mémoires scientifiques, publiés par J. L. Heiberg et G. H. Zeuthen. Tome I: Sciences exactes dans l'antiquité. (1876–1884.) Paris, Gauthier-Villars, 1912. 8vo. 19+466 pp. Fr. 15.00

THUE (A.). Ueber einen Zusammenhang zwischen den Gleichungen $Ax^3 + By^3 + Cz^3 = Dxyz$, $Px + Qy + Rz = 0$. (Videnskapsselskapets skrifter. I, 1911. Nr. 20.) Kristiania, Dybwad, 1912. 8vo. 20 pp. M. 1.00

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ZEUTHEN (G. H.). See TANNERY (P.).

II. ELEMENTARY MATHEMATICS.

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BERICHTE und Mitteilungen, veranlasst durch die internationale mathematische Unterrichtskommission. Nr. VII. Leipzig, Teubner, 1912. 8vo. pp. 89–127. M. 1.60

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PUBLIC SCHOOL examination papers in mathematics. Compiled by P. A. Openshaw. London, Bell, 1912. 8vo. 144 pp. 1s. 6d.

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LECTURES delivered at the celebration of the twentieth anniversary of the foundation of Clark University, Sept. 7–11, 1909. Volterra: Sur quelques progrès récents de la physique mathématique. E. Rutherford: The history of the alpha rays from radio-active substances. R. W. Wood: The optical properties of metallic vapors. C. Barus: Physical properties of the iron carbides. Worcester, Clark University, 1912. 8vo. 4+161 pp.

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